From forward modeling of acoustic wave equation towards to reverse time migration (rtm)

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Adquisition of seismic data

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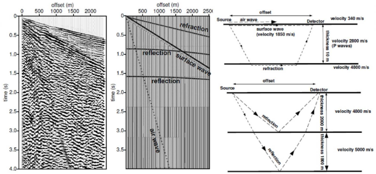

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Overview to seismic processing (Schuster, 2010 (book))

1. Filtering of noise and near-surface statics corrections

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2. Reassembly of common shot gather (CSG) traces into common midpoint gathers (CMG) where the source-receiver pair of each trace has the same midpoint location

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4. Stack the traces in the time-shifted CMG to form a single trace at the common midpoint position. This stacked trace approximates a ZO trace with a high signal-to-noise at that position

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5. Repeat steps 3-4 for all midpoint gathers to give the seismic section

- Migration (Schuster, 2010)
- Inversion (Schuster, 2010)
- RTM (Baysal *et. al.*, 1983)

$$[P(x,z,T+\Delta t) - P(x,z,T-\Delta t)]/2\Delta t = \dot{P}(x,z,T)$$

Exploding reflector model (Loewentgal *et. al.*, 1976)
Imaging Condition (Claerbout, 1985)

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Exploding reflector model (Loewenthal *et. al.*, 1976)
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- Migration (Schuster, 2010)
  Is defined as the process which takes the seismic section d(x,z,t) and moves the reflection events back to their origin at the interfaces

$$\mathbf{d} = \mathbf{L}(\mathbf{m})$$

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  Full wave inversion can be described as an iterative sequence of migrations, where the data residuals updated and migrated at each iteration to give the new model update, see

$$\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} + [\mathbf{L}^T_{(k)}\mathbf{L}_{(k)}]^{-1}\mathbf{L}^T_{(k)}\mathbf{d}^{(k)}$$

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Exploding reflector model (Loewentgal *et. al.*, 1976)
An approximation to a stacked section can be obtained in a single experiment by replacing the subsurface with a medium containing half the actual velocities in the earth, and by initiating explosive sources at time zero on all the reflecting boundaries Imaging Condition (Clearbout, 1985)

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$$\begin{aligned} \mathbf{P}(\vec{x},t) &= \mathbf{P}_s(\vec{x},t)\bigstar\mathbf{P}_r(\vec{x},t) \\ \mathbf{R}(\vec{x}) &= \mathbf{P}(\vec{x},t=0) \sim \mathbf{m} \end{aligned}$$

Acoustic limit of the elastic wave propagation

The consequences...

The consequences of this acoustic approximation include the restrictions to isotropic source radiation patterns and absence of the Rayleigh waves and P-to-S conversions. The acoustic approximatation is nevertheless, justificable when data analysis is restricted to the first-arriving P waves and when the seismic sources radiate little S wave energy (e.g. explosions) (Fichtner, 2011)[a]

[a] Page 14, Andreas Fichtner, "Full Seismic Waveform Modelling and Inversion", Advances in Geophysical and Environmental Mechanics and Mathematics, Springer(2011)

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Acoustic limit of the elastic wave propagation

- Elastic Equation of Motion

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$$
\begin{aligned}
\rho(\vec{x})\ddot{\vec{u}}(\vec{x},t) - \nabla.[\mathbb{C}(\vec{x}) : \nabla\vec{u}(\vec{x},t)] &= \vec{f}(\vec{x},t) \\
\sigma(\vec{x},t) &= \mathbb{C}(\vec{x}) : \nabla\vec{u}(\vec{x},t)
\end{aligned}
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- Discrete version of stress-velocity diffusion equations

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\mathbb{M}.\dot{\bar{\mathbf{v}}}(t) + \mathbb{K}_1.\bar{\mathbf{s}}(t) &= \bar{\mathbf{f}}(t) \\
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- Acoustic Limit

$$
\begin{aligned}
\rho(\vec{x})\partial_t\vec{u}(\vec{x},t) &= -\nabla p(\vec{x},t) \\
\frac{1}{\rho(\vec{x})c(\vec{x})^2}\partial_t p(\vec{x},t) &= -\nabla.\vec{u}(\vec{x},t)
\end{aligned}
$$

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Numerical Implementation (D. Kosloff and D. Kessler,198?)[1]

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- Pseudospectral Method (PS) via Finite Differences (FD) $(t, \vec{k})$

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Numerical Implementation (D. Kosloff and D. Kessler,198?)[1]

- Pseudospectral Method (PS) via Finite Differences (FD) $(t, \vec{k})$
  Diffusion Equations 2D $(\rho(\vec{x}) = 1, c(\vec{x}) = 1)$

$$\begin{aligned}
\partial_t p(\vec{x}, t) + \partial_x u(\vec{x}, t) + \partial_z v(\vec{z}, t) &= 0 \\
\partial_t u(\vec{x}, t) + \partial_x p(\vec{x}, t) &= 0 \\
\partial_t v(\vec{x}, t) + \partial_z p(\vec{x}, t) &= 0
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- Numerical error be follows from interplay between wevalength of the grid $(\triangle x, \triangle z)$, the order of the finite difference operators and the number of iterations N or $\triangle t$

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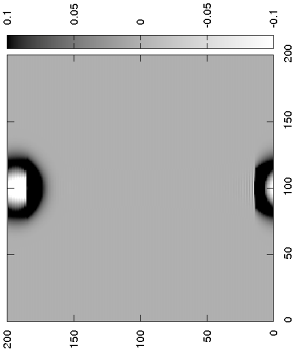
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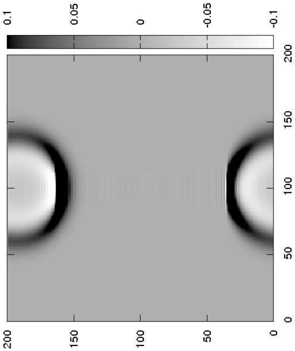

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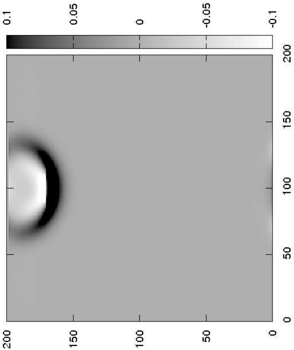

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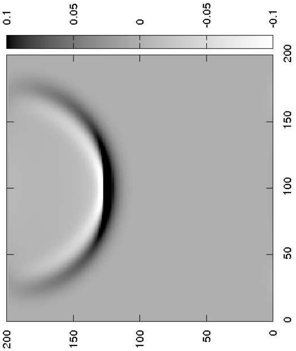

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Another works and conlusions

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1. Modeling in acoustic variable-density media by Fourier finite differences (Xiaolei Son, 2012)

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8. There is more . . .

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Conlusions

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2. From the elastic wave equation we derive stress-velocity diffusion system equations on which we take the acoustic limit restricting our data analysis
3. The Pseudospectral methods via finite differences are implemented in acoustic media as a trial platform and it offers interesting possibilities in inhomogeneous media

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