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Enhancing images in seismic migration is not an easy task

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Outline

Laguerre-Gauss transform

The Laguerre-Gauss transform of $I(x, y)$ is given by (Wang et al, 2006, [39], Guo et al, 2006, [15]):

$$\tilde{I}(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} LG(f_x, f_y) I(f_x, f_y) e^{2\pi i(f_x x, f_y y)} df_x df_y \quad (1)$$

where

$$LG(f_x, f_y) = (f_x + if_y)e^{-(f_x^2+f_y^2)/\omega^2} = \rho e^{-(\rho^2/\omega^2)} e^{i\beta} \quad (2)$$

$\rho = \sqrt{f_x^2 + f_y^2}$, $\beta = \tan^{-1}\left(\frac{f_y}{f_x}\right)$ are the polar coordinates in the spatial frequency domain.

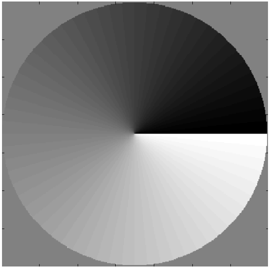
Laguerre-Gauss transform

$$\tilde{I}(x, y) = |\tilde{I}(x, y)| e^{i\theta(x,y)} = I(x, y) * LG(x, y) \quad (3)$$

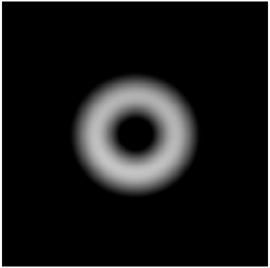
From (2) we obtain

$$\begin{aligned} LG(x, y) &= \mathscr{F}^{-1}\{LG(f_x, f_y)\} = (i\pi^2\omega^4)(x + iy)e^{-\pi^2\omega^2(x^2+y^2)} \\ &= (i\pi^2\omega^4)[re^{-\pi^2 r^2\omega^2} e^{i\alpha}] \end{aligned} \quad (4)$$

Laguerre-Gauss transform



Spiral phase function

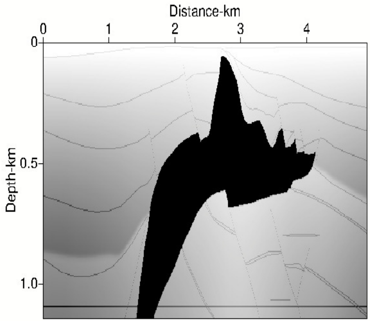


Toroidal amplitude

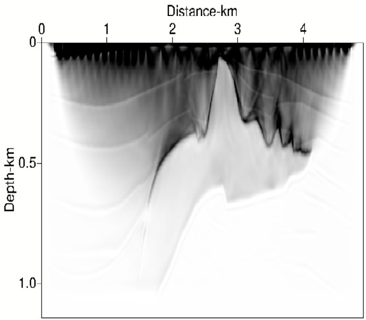
Figure: Laguerre-Gauss Filter (Wang et al, 2006, [39])

$2D$ SEG EAGE model

Velocity model

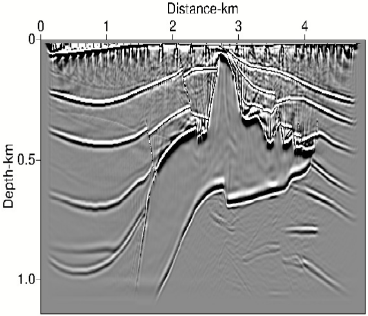


Cross correlation image

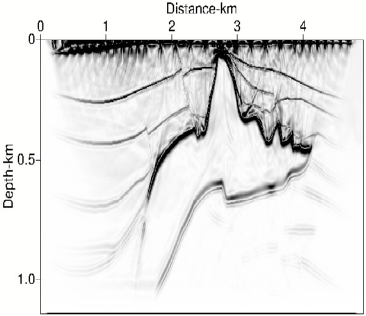


$2D$ **SEG EAGE model**

Laplacian image

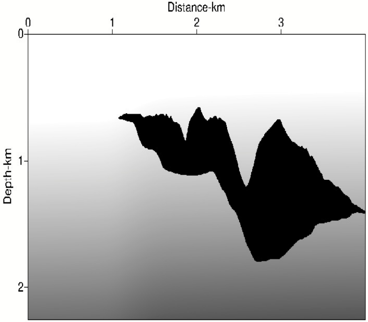


Laguerre-Gauss image

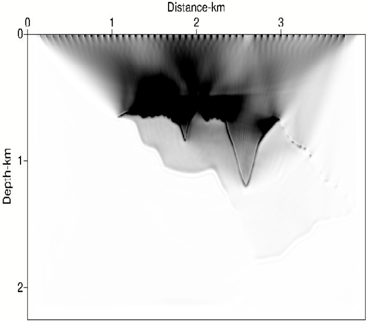


$2D$ **Sigsbee2A model**

Velocity model

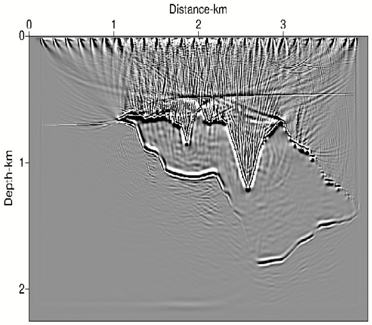


Cross correlation image

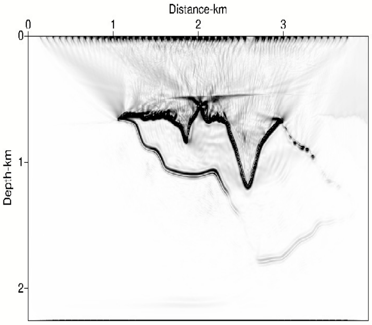


$2D$ **Sigsbee2A model**

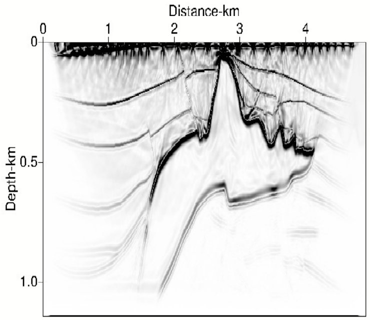
Laplacian image



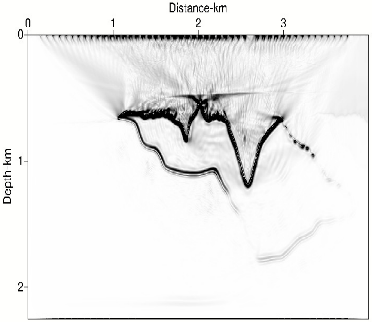
Laguerre-Gauss image



LG SEG EAGE image

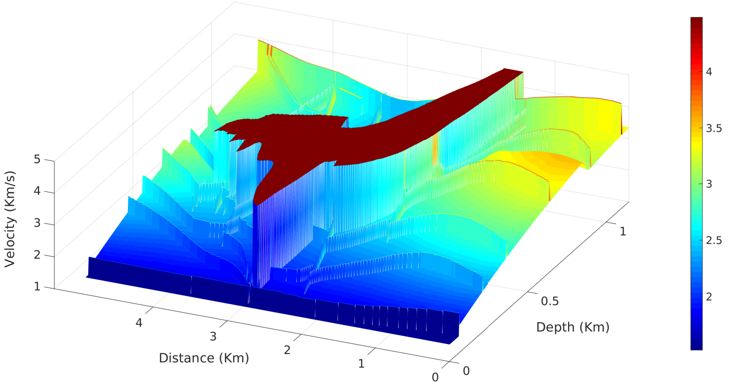


LG Sigsbee2A image



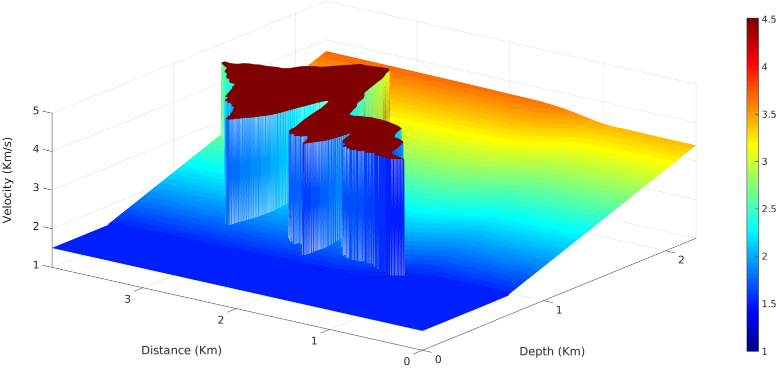
2*D* SEG EAGE model

Velocity model



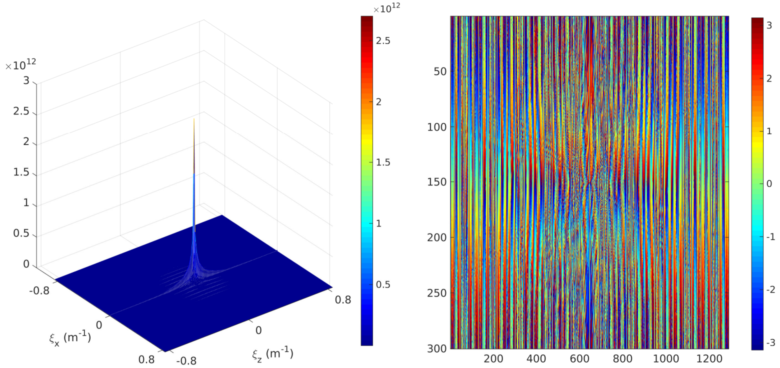
$2D$ **Sigsbee2A model**

Velocity model



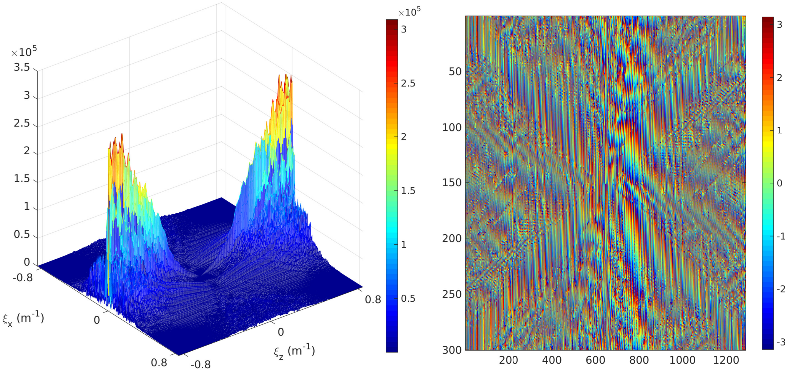
$2D$ SEG EAGE model

Amplitude and phase spectra of cross correlation image



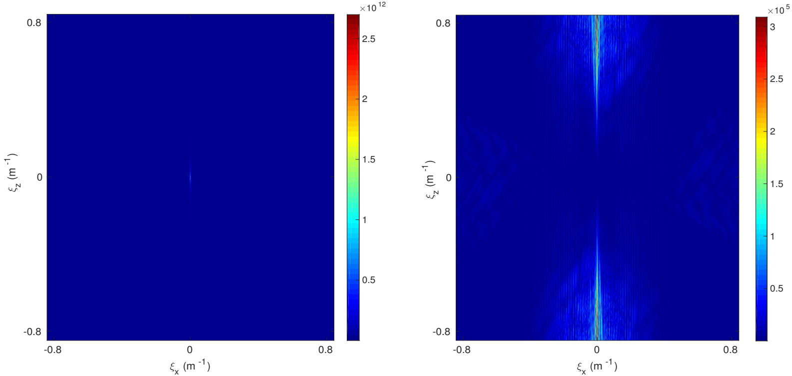
2*D* **SEG EAGE model**

Amplitude and phase spectra of Laguerre-Gauss image



$2D$ **SEG EAGE model**

Amplitude spectrum of cross correlation and Laguerre-Gauss images



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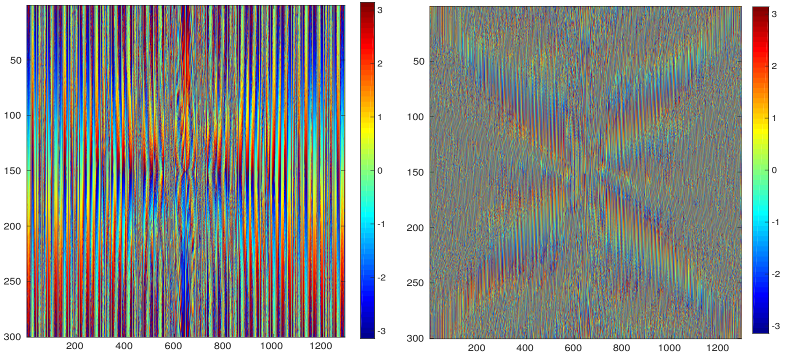
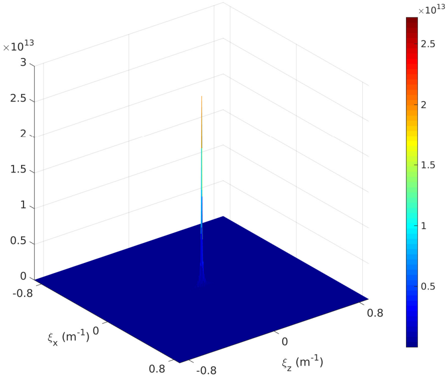
$2D$ **SEG EAGE model**

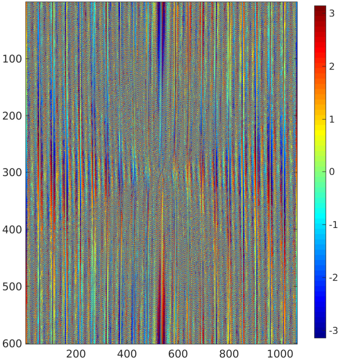
Phase spectrum of cross correlation and Laguerre-Gauss images



$2D$ Sigsbee2A model

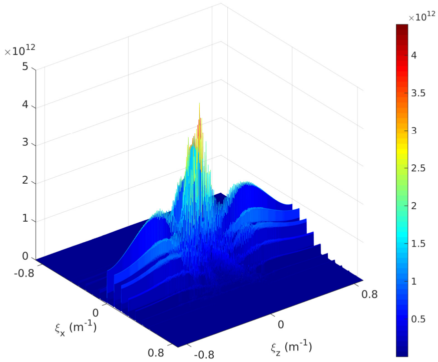
Amplitude and phase spectra of cross correlation image

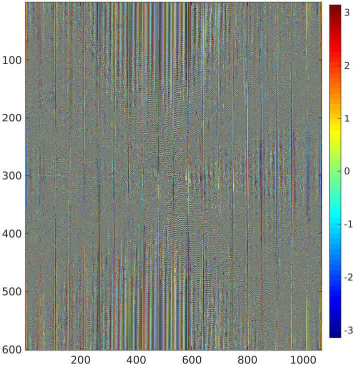




2D **Sigsbee2A model**

Amplitude and phase spectra of Laguerre-Gauss image





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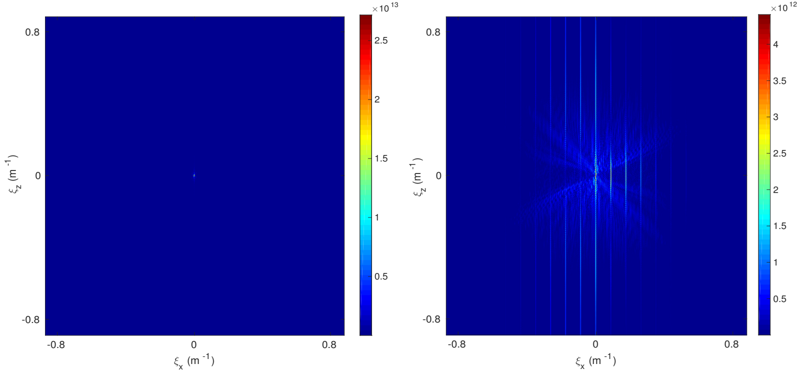
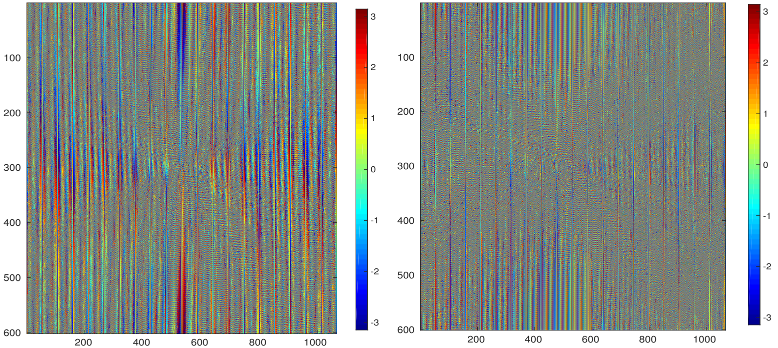
$2D$ **Sigsbee2A model**

Amplitude spectrum of cross correlation and Laguerre-Gauss images



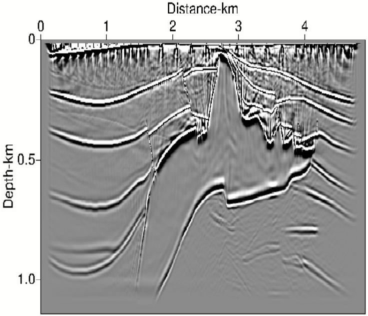
$2D$ Sigsbee2A model

Phase spectrum of cross correlation and Laguerre-Gauss images

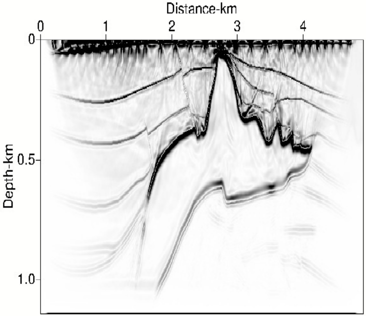


$2D$ **SEG EAGE model**

Laplacian image

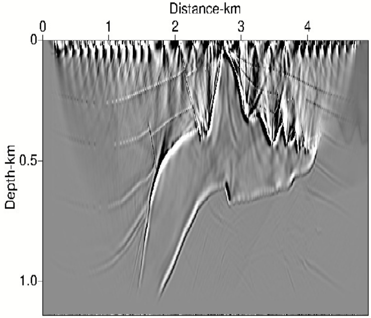


Laguerre-Gauss image

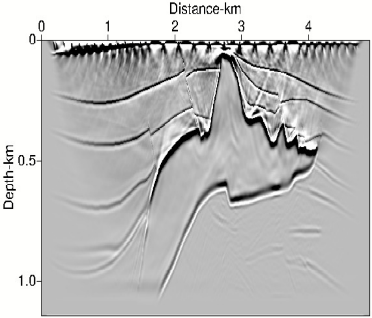


$2D$ SEG EAGE model

Real part of LG image

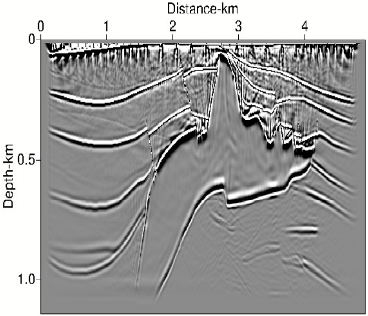


Imaginary part of LG image

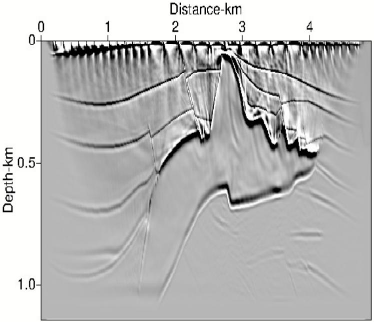


$2D$ SEG EAGE model

Laplacian image

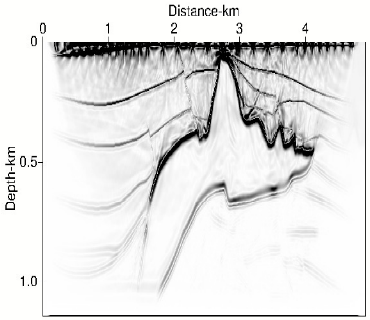


Real + imaginary parts



$2D$ SEG EAGE model

Laguerre-Gauss image



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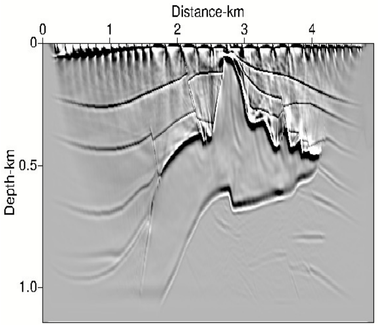
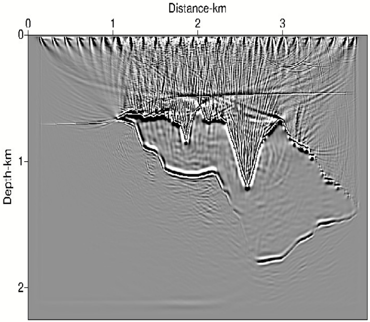
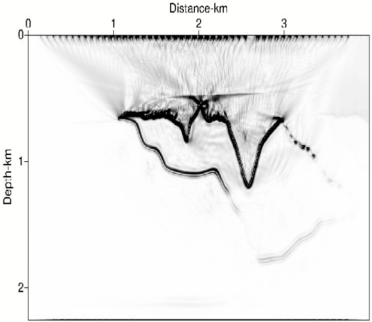


2D **Sigsbee2A model**

Laplacian image

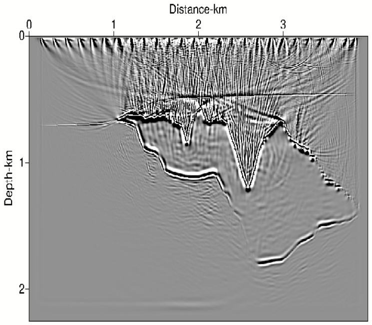


Laguerre-Gauss image

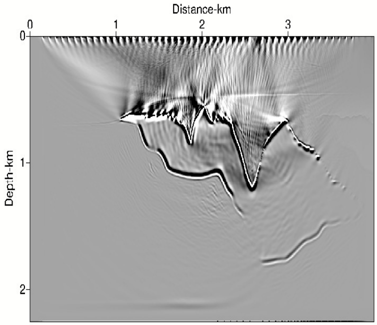


$2D$ Sigsbee2A model

Laplacian image

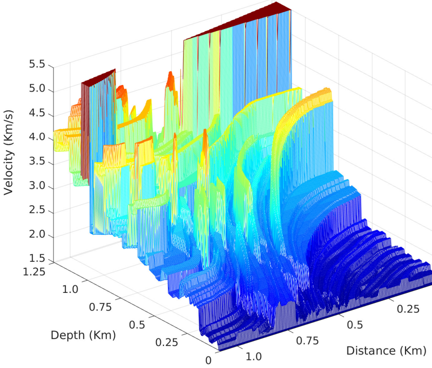


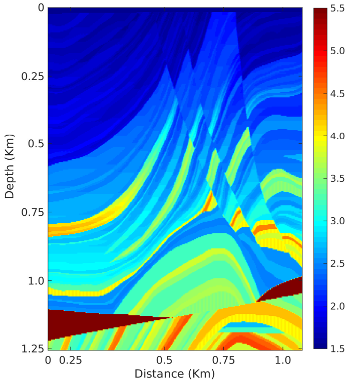
Real + imaginary parts



2*D* Marmoussi model

Velocity model





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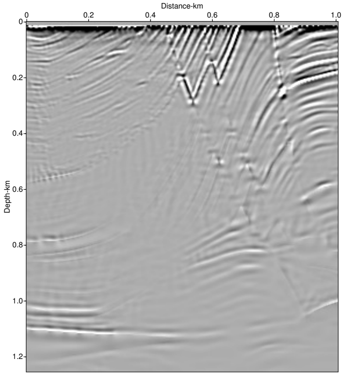
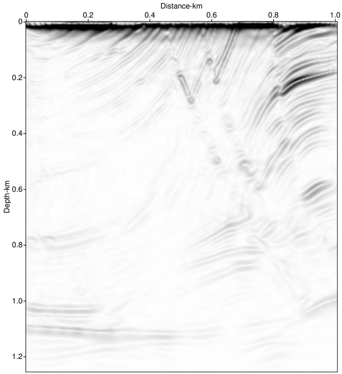
2*D* Marmoussi model

Laplacian image

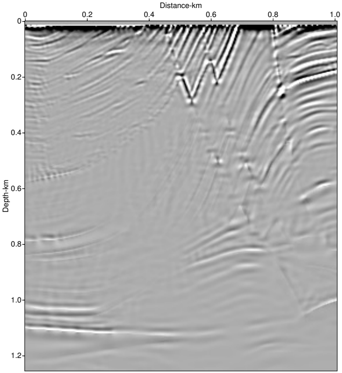


Laguerre-Gauss image

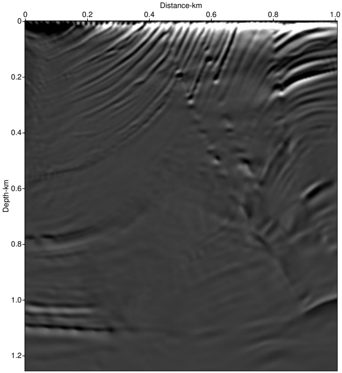


$2D$ **Marmoussi model**

Laplacian image



Laguerre-Gauss image



Future work

- Analyze why the sum of real and imaginary parts of pseudo complex field obtained by Laguerre-Gauss filtering improves the cross correlation image.
- Find the relationship between the low and high frequency filtered values and ω parameter.

Future work

- Perform a mathematical analysis of the source and receiver wavefields obtained in RTM to study its effects in the cross correlation imaging condition and the illumination maps.
- Analyze of singularity spectrum of the seismograms, and the source and receiver wavefields in order to obtain additional information to use in RTM imaging.

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